OTS: 60-31,189

JPRS: 3309

2 June 1960

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- USSR -

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19990430 135

Distributed by:

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Price: $0.50

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JPRS: 3309

CSO: 3642-D

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[Following is a translation of an article by E. I. Andrianin in Zhurnal Tekhnicheskoy Fiziki (Journal of Technical Physics), Vol. XXIX, No. 11, Moscow-Leningrad, November 1959, pages 1368-1372.]

The article investigates the propagation of a nonprogressing heat wave radiating energy from a front. It examines the case when the travel of radiation in a cold gas is large for all frequencies below a certain critical frequency ω_*, and small for higher frequencies. The travel of quanta in the heated region is assumed to be much shorter than the radius of the wave front, so that the radiant transfer of energy occurs by thermal conductivity.

When a heat wave of high temperature propagates through a gas its molecules split up into atoms, the latter ionizing to form plasma with a high radiant heat conductivity (1). It can sometimes be considered that the radiation travel in a plasma is much shorter than the radius of the wave front.

The coefficient of radiation absorption in a cold gas greatly depends on the frequency of quanta and rapidly approaches zero for long wavelengths. Therefore, as Kompaneyets showed, it can be considered that only those quanta leave the eave front whose frequency is lower than a certain value ω_*. The problem of a progressing heat wave propagating from a point in which there is a sudden liberation of a quantity of energy Q, assuming the internal energy and the heat transfer coefficient to be exponentially dependent on temperature but without taking into account the radiation of energy from the front, was investigated by Zel'dovich and Kompaneyets (2).

Let us examine by the approximate method a similar problem, taking into account the radiation of energy from the wave front and applying the relationships of moments which were studied in detail by Barenblatt (3) in the theory of the filtration of a gas in a porous medium, and by Loytsyanskiy (4) in the theory of a boundary layer.

We shall use for our study a particular example, when the internal energy of the gas is proportional to the temperature, and the length of the free travel of radiation is a function of temperature in accordance with the exponential law $l = l_0 T^q$. Let us note, however, that this same method may be used to study the problem when there is an arbitrary dependence of the internal energy and radiation travel on temperature.

We shall consider that the cold gas passes all frequencies lower than ω_* and detains frequencies higher than ω_*. We shall disregard the shielding effect on radiation due to the heating of the prefrontal zone. The condition for energy balance at the wave front can then be written thus

$$aT_{\phi}\dot{r}_{\phi} + c_0 \left(\frac{\partial T^k}{\partial r}\right)_{\phi} = -S(T_{\phi}), \qquad k = q + 4,$$

$$S = \int_0^{\omega_*} \frac{h\omega^3 d\omega}{4\pi^2 c^2 \left(e^{\frac{h\omega}{kT_{\phi}}} - 1\right)}, \qquad c_0 = \frac{16\sigma l_0}{3k}, \tag{1}$$

$h = 6.625\cdot 10^{-27}$ erg/sec; $k = 1.38\cdot 10^{-16}$ erg/degree; a — heat capacity per unit volume; c — velocity of light, the index "ϕ." signifies that values refer to the wave front. In a number of cases, S can approximate the exponential dependence. If $h\omega \ll kT_{\phi}$, then, resolving the exponent into the series (Rayleigh-Jeans formula), we obtain

$$S = \frac{kT_{\phi}\,\omega_*^3}{12\pi^2 c^2}.$$

If $h\omega \gg kT_{\phi}$, then $S \approx T_{\phi}^4$. The maximum flow of energy is obtained in the case $\omega_* = \infty$ (all frequencies luminesce [vysvechivayutsya]) then $S = \sigma T^4$. $\sigma = 5.67\cdot 10^{-5}$ erg/cm^2.sec.degree is the Stefan-Boltzmann constant. Besides (1), we must still fulfill the condition so that the flow of heat in the center of the wave is equal to zero

$$r^2 \frac{\partial T^k}{\partial r}\bigg|_{r=0} = 0. \tag{2}$$

We shall write the equation for heat transfer in a quiescent medium

$$a \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \chi \frac{\partial 4 \sigma T^4}{c \partial r}, \quad \chi = \frac{cl(T)}{3}. \qquad (3)$$

Equation (3) is equivalent to an infinite number of relationships which may be obtained by integrating this equation, previously multiplied by r^m (m 2, 3, 4...). Applyijg the method of moments, we shall satisfy equation (3), having required approximate fulfillment of only two integral relationships, corresponding to m = 2 and 3. Employing (1) and (3), we obtain the equalities

$$4\pi a \int_0^{r_\Phi} Tr^2 dr = Q_0 - 4\pi \int_0^t S(T_\Phi) r_\Phi^2 dt, \qquad (4)$$

$$\frac{d}{dt} a \int_0^{r_\Phi} r^3 T dr = - r_\Phi^3 S(T_\Phi) - c_0 r_\Phi^2 T_\Phi^{(k)} + 2c_0 \int_0^{r_\Phi} rT^k dr. \qquad (5)$$

Despite the fact that out of an infinite number we have satisfied only two equalities (4) and (5), we can hope for good agreement between approximate and accurate solutions, since the temperature behind the wave front changes evenly and monotonically.

Since the law governing the distribution of temperature behind the wave front is basically determined by the thermal conductivity, we shall search for a solution in a form similar to the progressing problem

$$T = T_0(t) \left[1 - \left(\frac{rA}{r_\Phi}\right)^2\right]^{\frac{1}{k-1}}, \qquad (6)$$

where, however, $T_0(t)$, $A(t)$, $r_\Phi(t)$ are previously unknown functions satisfying equalities (1), (4) and (5). Applying (6) and combining (2), (4) and (5), we arrive at a system of equations

$$r_{\Phi} \frac{\dot{A}}{A} + \frac{S}{aT_{\Phi}} = 0, \quad \frac{\dot{T}_{\Phi}}{T_0} + 3\left(\frac{\dot{r}_{\Phi}}{r_{\Phi}} - \frac{\dot{A}}{A}\right) = 0,$$

$$\frac{2c_0kA^2}{a(k-1)r_{\Phi}^2} T_0^{k-1} + \frac{\dot{T}_0}{3T_0} = 0, \quad T_{\Phi} = T_0(1 - A^2)^{\frac{1}{k-1}}. \tag{7}$$

Equation (7) is easily integrated. In the case $S = \mu T_{\Phi}^{n+1}$, we obtain

$$x^{3k-1} = C_1 t + C_2, \quad T_0^{k-1} = \frac{a\,(k-1)C_1}{2kc_0(3k-1)} x^{3(1-k)}, \quad x = \frac{r_{\Phi}}{A},$$

$$Bx^{3k-3n-2} = \int_A^1 (1 - A^2)^{\frac{n}{1-k}}\, dA + C_3. \tag{8}$$

The constants C_1, C_2 and C_3 are determined from initial conditions.

If initially the process differs from that described by us, but at the moment of time t_0 there is formed a heat wave with radius r_0 and a distribution of temperature

$$T(t_0, r) = T_{00}\left[1 - \left(\frac{rA_0}{r_0}\right)^2\right]^{\frac{1}{k-1}},$$

then the initial conditions for determining the three constants T_{00}, r_0, A_0, will be

$$T_0(r_0, t_0) = T_{00}, \quad r_{\Phi}(t_0) = r_0, \quad A(t_0) = A_0.$$

If liberation of energy occurs at a point, then during the initial moment solution must become progressing. Therefore, we must assume

$$C_2 = C_3 = 0, \quad C_1 = \frac{c_0}{a} \left[\frac{Q_0}{2\pi\gamma(k)} \right]^{k-1},$$

$$\gamma(k) = \left[\frac{k-1}{2k(3k-1)} \right]^{\frac{1}{k-1}} \Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{k}{k-1}\right) \left[\Gamma\left(\frac{3}{2} + \frac{k}{k-1}\right) \right]^{-1}. \tag{9}$$

Let, for example, $n = 0$, then from formulas (8) and (9) we obtain the series of expressions

$$A_* = \frac{3k-2}{3k-1}, \quad r_{\phi *} = \left[\frac{C_1}{(3k-1)\lambda} \right]^{\frac{1}{3k-2}} \left(\frac{3k-2}{3k-1}\right)^{\frac{3k-1}{3k-2}},$$

$$\dot{r}_{\phi.*} = r_1 = A_1 = 0, \quad \dot{r}_1 = -\lambda, \quad A_2 = \frac{3(k-1)}{3k-1},$$

$$T_{01} = T_{\phi.1} = \left[\frac{a(k-1)}{2k(3k-1)c_0} \right] \left[\frac{\lambda(3k-1)}{3k-2} \right]^{\frac{3}{3k-2}} C_1^{\frac{1}{(3k-2)(k-1)}}, \tag{10}$$

$$\frac{T_{\phi.max}}{T_{\phi.1}} = \left(\frac{2}{3k-1} \right)^{\frac{3k-1}{(3k-2)(k-1)}} (3k-2)^{\frac{1}{k-1}}, \quad \lambda = \frac{\mu}{a}.$$

Here, the values corresponding to the maximum radius of the front of the heat wave are marked by an asterisk, and the values at the moment when all energy is radiated -- by the index "1"; $T_{\phi.max}$ is the maximum temperature at the wave front. A_2 is the value of A corresponding to this temperature. The formulas (10) describe the quite obvious picture of heat wave propagation with consideration for luminescence in the motionless gas.

At first the radius of the wave front increases in time, reaching its maximum value r_ϕ, after which the dimensions of the heated region begin to shrink and the radius of the front again decreases to zero, at which time all the original energy is radiated [vysvechivayetsya]. The temperature at the wave front at first increases from zero to maximum $T_{\phi\ max}$; this is reached, however, after the radius of the front begins to decrease.

The extreme values of all quantities are easily found from the formulas (8) as well as for any arbitrary value of n. Thus, for example, for A_* and A_1 we have

$$(3k - 3N - 2) \int_1^{A_*} Z(A)dA + Z(A_*)A_* = 0, \quad Z = (1 - A^2)^{\frac{n}{1-k}},$$

$$2A_2 \int_{A_2}^{1} Z(A)dA + (1 - A^2) \frac{3(1 - k)}{3k - 3n - 2} = 0. \qquad (11)$$

We note that from the formulas (8), in the case when the liberation of heat occurs at a point, it follows that the temperature in the center changes according to the progressing law, and that the temperature profile coincides with the progressing profile, which is clipped off at a distance from the center equal to $r_{\phi} = r_{\phi 0}A$ ($r_{\phi 0}$ is the radius of the wave for the progressing problem).

Solution of equation (3) could be sought in another form

$$T = \left[\frac{a\,(k - 1)}{2k(3k - 1)c_0}\, C^{\frac{2}{3k-1}}\right]^{\frac{1}{k-1}} t^{-\frac{3}{3k-1}}$$

$$\left[1 - \left(\frac{rA}{r_{\phi}}\right)^2\right]^{\alpha(t)},$$

determining $r_{\phi}(t)$, $A(t)$ and $\alpha(t)$ similarly as above from equations (1), (4) and (5).

Despite the fact that, mathematically, the formulas (8) describe the propagation of the heat wave in all stages right up to complete radiative dissipation at a certain distance from the origin of the coordinates the solution already loses its physical meaning by virtue of the emergent motion of the gas. Since the heat wave slows down quite abruptly, we can therefore determine qualitatively the limit of applicability of our solution, as indicated by Kompaneyets (5), from the conditions for equality between the velocity of the wave front and the velocity of sound in its center. Energy transfer

proceeds further by gas-dynamical means, since the elementary physical requirements are fulfilled so that the small disturbances overtake the front of pronounced discontinuity. Let the velocity of sound be given by the formula

$$C_* = b\, T_4^{1/2}, \qquad T_4 = \frac{3Q}{4\pi a r^3}.$$

Since the temperature profile approaches a plateau owing to the intense heat transfer, T_4 is close to T_0; we then obtain the formula which determines A_3 from the condition $C_* = r_\phi$.

$$\left[\frac{A_3 Z}{(3k - 3n - 2)\, I} - 1 \right] \frac{1}{ZA^{\frac{3(2n-1)}{2}}\; I^{\frac{3(2n-1)}{2(3k-3n-2)}}} = \beta, \qquad (12)$$

$$\beta = \frac{b}{\lambda} \left(\frac{4\pi}{3Q} \right)^{\frac{2n-1}{2}} B^{\frac{3(1-2n)}{2(3k-3n-2)}}, \qquad I = \int_{A_3}^{1} Z\,dA.$$

It is interesting to note that the values for A_3 approach unity even for the limiting case when $\beta = 0$. These values are determined from the conditions when the expressions contained in brackets in equation (12) are reduced to zero. In the case $n = 0$, expression (12) becomes simple, and the maximum value for A_3, corresponding to $\beta = 0$, is determined by the formula

$$A_{3max} = A_* = \frac{3k - 2}{3k - 1}. \qquad (13)$$

The velocity of the front of the heat wave at this time becomes equal to zero. For $\beta > 0$, the values for A_3, calculated from formula (12), lie still closer to unity. For example, at $n = 0$, $k = 6$, $\beta = 200$, $A \approx 1 - 2.5 \cdot 10^{-4}$.

Making use of the proximity of A_3 to unity, expression (12) can be linearized, after which we obtain

$$A_{3max} = 1 - \frac{k - n - 1}{k - n - 1 + (k-1)(3k - 3n - 2)}. \qquad (14)$$

Knowing the temperature distribution along the radius at each moment, it is easy to determine the share of radiated energy

$$\frac{Q - Q}{Q} = \frac{2}{\gamma} \left[\frac{k - 1}{2k(3k - 1)} \right]^{\frac{1}{k-1}} \int_0^A y^2 (1 - y^2)^{\frac{1}{k-1}} dy. \qquad (15)$$

The results of computations by this formula for the case $k = 3$ and $k = 6$ are shown in the figure. We notice that the amount of radiated energy at a given distance depends on the law of forced luminescence only through the value A. Therefore, at $k = 6$, $n = 0$, $A \geqslant A_{3max}$, $\frac{\Delta Q}{Q} \geqslant 0.9$; at $k = 3$, $\frac{\Delta Q}{Q} \geqslant 0.95$.

For the case when

$$n = 0, \; k = 6, \; \beta = 200, \; \frac{\Delta Q}{Q} \simeq 1 - 5 \cdot 10^{-4}.$$

From the above it follows that the loss of energy through radiation from a wave front up to the moment when $r_\phi = c_0$ is small, even if radiation is very intense. Physically, this results from the fact that an increase in radiation retards the wave front, and the motion of the gas is found to be substantial at distances closer to the origin of coordinates.

In conclusion, let us note that in the case when the length of the free travel of radiation is commensurate with the dimensions of the heated region, an approximate solution can be found by the method of moments.

The author extends his sincere thanks to A. S. Kompaneyets for his valuable assistance.

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Submitted for Publication
January 30, 1959

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